

Table 5A.1 Eight-Run 2^{k-p} Fractional Factorial Designs ($k - p = 3$)

Number of Factors k	Fraction and Resolution	Design Generators	Clear Effects
4 ^a	2_{IV}^{4-1}	4 = 123	1, 2, 3, 4
5 ^b	2_{III}^{5-2}	4 = 12, 5 = 13	None
6	2_{III}^{6-3}	4 = 12, 5 = 13, 6 = 23	None
7	2_{III}^{7-4}	4 = 12, 5 = 13, 6 = 23, 7 = 123	None

^aThe aliases are 1 = 234, 2 = 134, 3 = 124, 4 = 123, 12 = 34, 13 = 24, 14 = 23.

^bThe aliases are 1 = 24 = 35 = 12345, 2 = 14 = 345 = 1235, 3 = 15 = 245 = 1234, 4 = 12 = 235 = 1345, 5 = 13 = 234 = 1245, 23 = 45 = 125 = 134, 25 = 34 = 123 = 145.

Table 5A.2 Sixteen-Run 2^{k-p} Fractional Factorial Designs ($k - p = 4$) (k is the number of factors and F & R is the fraction and resolution)

k	F & R	Design Generators	Clear Effects
5	2_V^{5-1}	5 = 1234	All five main effects, all 10 2fi's
6	2_{IV}^{6-2}	5 = 123, 6 = 124	All six main effects
6 ^a	2_{III}^{6-2}	5 = 12, 6 = 134	3, 4, 6, 23, 24, 26, 35, 45, 56
7	2_{IV}^{7-3}	5 = 123, 6 = 124, 7 = 134	All seven main effects
8	2_{IV}^{8-4}	5 = 123, 6 = 124, 7 = 134, 8 = 234	All eight main effects
9	2_{III}^{9-5}	5 = 123, 6 = 124, 7 = 134, 8 = 234, 9 = 1234	None
10	2_{III}^{10-6}	5 = 123, 6 = 124, 7 = 134, 8 = 234, 9 = 1234, $t_0 = 34$	None
11	2_{III}^{11-7}	5 = 123, 6 = 124, 7 = 134, 8 = 234, 9 = 1234, $t_0 = 34, t_1 = 24$	None
12	2_{III}^{12-8}	5 = 123, 6 = 124, 7 = 134, 8 = 234, 9 = 1234, $t_0 = 34,$ $t_1 = 24, t_2 = 14$	None
13	2_{III}^{13-9}	5 = 123, 6 = 124, 7 = 134, 8 = 234, 9 = 1234, $t_0 = 34,$ $t_1 = 24, t_2 = 14, t_3 = 23$	None
14	2_{III}^{14-10}	5 = 123, 6 = 124, 7 = 134, 8 = 234, 9 = 1234, $t_0 = 34,$ $t_1 = 24, t_2 = 14, t_3 = 23, t_4 = 13$	None
15	2_{III}^{15-11}	5 = 123, 6 = 124, 7 = 134, 8 = 234, 9 = 1234, $t_0 = 34,$ $t_1 = 24, t_2 = 14, t_3 = 23,$ $t_4 = 13, t_5 = 12$	None

^aIts aliases are given in (5.3) of Section 5.2.

Table 5A.3 Thirty-Two Run 2^{k-p} Fractional Factorial Designs ($k - p = 5$, $6 \leq k \leq 16$) (k is the number of factors and F & R is the fraction and resolution)

k	F & R	Design Generators	Clear Effects
6	2_{VI}^{6-1}	6 = 12345	All six main effects, all 15 2fi's
7	2_{IV}^{7-2}	6 = 123, 7 = 1245	All seven main effects, 14, 15, 17, 24, 25, 27, 34, 35, 37, 45, 46, 47, 56, 57, 67
8	2_{IV}^{8-3}	6 = 123, 7 = 124, 8 = 1345	All eight main effects, 15, 18, 25, 28, 35, 38, 45, 48, 56, 57, 58, 68, 78
9	2_{IV}^{9-4}	6 = 123, 7 = 124, 8 = 125, 9 = 1345	All nine main effects, 19, 29, 39, 49, 59, 69, 79, 89
9	2_{IV}^{9-4}	6 = 123, 7 = 124, 8 = 134, 9 = 2345	All nine main effects, 15, 19, 25, 29, 35, 39, 45, 49, 56, 57, 58, 59, 69, 79, 89
10	2_{IV}^{10-5}	6 = 123, 7 = 124, 8 = 125, 9 = 1345, $t_0 = 2345$	All 10 main effects
10	2_{III}^{10-5}	6 = 12, 7 = 134, 8 = 135, 9 = 145, $t_0 = 345$	3, 4, 5, 7, 8, 9, t_0 , 23, 24, 25, 27, 28, 29, $2t_0$, 36, 46, 56, 67, 68, 69, $6t_0$
11	2_{IV}^{11-6}	6 = 123, 7 = 124, 8 = 134, 9 = 125, $t_0 = 135$, $t_1 = 145$	All 11 main effects
11	2_{III}^{11-6}	6 = 12, 7 = 13, 8 = 234, 9 = 235, $t_0 = 245$, $t_1 = 1345$	4, 5, 8, 9, t_0 , t_1 , 14, 15, 18, 19, $1t_0$, $1t_1$
12	2_{IV}^{12-7}	6 = 123, 7 = 124, 8 = 134, 9 = 234, $t_0 = 125$, $t_1 = 135$, $t_2 = 145$	All 12 main effects
12	2_{III}^{12-7}	6 = 12, 7 = 13, 8 = 14, 9 = 234, $t_0 = 235$, $t_1 = 245$, $t_2 = 1345$	5, 9, t_0 , t_1 , t_2 , 15, 19, $1t_0$, $1t_1$, $1t_2$
13	2_{IV}^{13-8}	6 = 123, 7 = 124, 8 = 134, 9 = 234, $t_0 = 125$, $t_1 = 135$, $t_2 = 235$, $t_3 = 145$	All 13 main effects
14	2_{IV}^{14-9}	6 = 123, 7 = 124, 8 = 134, 9 = 234, $t_0 = 125$, $t_1 = 135$, $t_2 = 235$, $t_3 = 145$, $t_4 = 245$	All 14 main effects
15	2_{IV}^{15-10}	6 = 123, 7 = 124, 8 = 134, 9 = 234, $t_0 = 125$, $t_1 = 135$, $t_2 = 235$, $t_3 = 145$, $t_4 = 245$, $t_5 = 345$	All 15 main effects
16	2_{IV}^{16-11}	6 = 123, 7 = 124, 8 = 134, 9 = 234, $t_0 = 125$, $t_1 = 135$, $t_2 = 235$, $t_3 = 145$, $t_4 = 245$, $t_5 = 345$, $t_6 = 12345$	All 16 main effects

Table 5A.4 Thirty-Two Run 2^{k-p} Fractional Factorial Designs ($k - p = 5$, $17 \leq k \leq 31$) (k is the number of factors and F & R is the fraction and resolution)

k	F & R	Design Generators
17	2_{III}^{17-12}	$6 = 12, 7 = 13, 8 = 14, 9 = 234, t_0 = 1234, t_1 = 15, t_2 = 235, t_3 = 1235, t_4 = 245, t_5 = 1245, t_6 = 345, t_7 = 1345$
18	2_{III}^{18-13}	$6 = 12, 7 = 13, 8 = 23, 9 = 14, t_0 = 234, t_1 = 1234, t_2 = 15, t_3 = 235, t_4 = 1235, t_5 = 245, t_6 = 1245, t_7 = 345, t_8 = 1345$
19	2_{III}^{19-14}	$6 = 12, 7 = 13, 8 = 23, 9 = 14, t_0 = 24, t_1 = 234, t_2 = 1234, t_3 = 15, t_4 = 235, t_5 = 1235, t_6 = 245, t_7 = 1245, t_8 = 345, t_9 = 1345$
20	2_{III}^{20-15}	$6 = 12, 7 = 13, 8 = 23, 9 = 14, t_0 = 24, t_1 = 234, t_2 = 1234, t_3 = 15, t_4 = 25, t_5 = 235, t_6 = 1235, t_7 = 245, t_8 = 1245, t_9 = 345, u_0 = 1345$
21	2_{III}^{21-16}	$6 = 12, 7 = 13, 8 = 23, 9 = 14, t_0 = 24, t_1 = 234, t_2 = 1234, t_3 = 15, t_4 = 25, t_5 = 235, t_6 = 1235, t_7 = 245, t_8 = 1245, t_9 = 345, u_0 = 1345, u_1 = 12345$
22	2_{III}^{22-17}	$6 = 12, 7 = 13, 8 = 23, 9 = 14, t_0 = 24, t_1 = 134, t_2 = 234, t_3 = 1234, t_4 = 15, t_5 = 25, t_6 = 135, t_7 = 235, t_8 = 1235, t_9 = 145, u_0 = 245, u_1 = 1345, u_2 = 2345$
23	2_{III}^{23-18}	$6 = 12, 7 = 13, 8 = 23, 9 = 14, t_0 = 24, t_1 = 134, t_2 = 234, t_3 = 1234, t_4 = 15, t_5 = 25, t_6 = 135, t_7 = 235, t_8 = 1235, t_9 = 145, u_0 = 245, u_1 = 1245, u_2 = 345, u_3 = 1345$
24	2_{III}^{24-19}	$6 = 12, 7 = 13, 8 = 23, 9 = 14, t_0 = 24, t_1 = 134, t_2 = 234, t_3 = 1234, t_4 = 15, t_5 = 25, t_6 = 135, t_7 = 235, t_8 = 1235, t_9 = 145, u_0 = 245, u_1 = 1245, u_2 = 345, u_3 = 1345, u_4 = 2345$
25	2_{III}^{25-20}	$6 = 12, 7 = 13, 8 = 23, 9 = 123, t_0 = 14, t_1 = 24, t_2 = 124, t_3 = 34, t_4 = 134, t_5 = 15, t_6 = 25, t_7 = 125, t_8 = 35, t_9 = 135, u_0 = 245, u_1 = 1245, u_2 = 345, u_3 = 1345, u_4 = 2345, u_5 = 12345$
26	2_{III}^{26-21}	$6 = 12, 7 = 13, 8 = 23, 9 = 123, t_0 = 14, t_1 = 24, t_2 = 124, t_3 = 34, t_4 = 134, t_5 = 234, t_6 = 15, t_7 = 25, t_8 = 125, t_9 = 35, u_0 = 135, u_1 = 245, u_2 = 1245, u_3 = 345, u_4 = 1345, u_5 = 2345, u_6 = 12345$
27	2_{III}^{27-22}	$6 = 12, 7 = 13, 8 = 23, 9 = 123, t_0 = 14, t_1 = 24, t_2 = 124, t_3 = 34, t_4 = 134, t_5 = 234, t_6 = 15, t_7 = 25, t_8 = 125, t_9 = 35, u_0 = 135, u_1 = 235, u_2 = 145, u_3 = 245, u_4 = 1245, u_5 = 345, u_6 = 1345, u_7 = 2345$
28	2_{III}^{28-23}	$6 = 12, 7 = 13, 8 = 23, 9 = 123, t_0 = 14, t_1 = 24, t_2 = 124, t_3 = 34, t_4 = 134, t_5 = 234, t_6 = 15, t_7 = 25, t_8 = 125, t_9 = 35, u_0 = 135, u_1 = 235, u_2 = 145, u_3 = 245, u_4 = 1245, u_5 = 345, u_6 = 1345, u_7 = 2345, u_8 = 12345$

Table 5A.4 (Continued)

<i>k</i>	F & R	Design Generators
29	2^{29-24}_{III}	6 = 12, 7 = 13, 8 = 23, 9 = 123, $t_0 = 14$, $t_1 = 24$, $t_2 = 124$, $t_3 = 34$, $t_4 = 134$, $t_5 = 234$, $t_6 = 1234$, $t_7 = 15$, $t_8 = 25$, $t_9 = 125$, $u_0 = 35$, $u_1 = 135$, $u_2 = 235$, $u_3 = 145$, $u_4 = 245$, $u_5 = 1245$, $u_6 = 345$, $u_7 = 1345$, $u_8 = 2345$, $u_9 = 12345$
30	2^{30-25}_{III}	6 = 12, 7 = 13, 8 = 23, 9 = 123, $t_0 = 14$, $t_1 = 24$, $t_2 = 124$, $t_3 = 34$, $t_4 = 134$, $t_5 = 234$, $t_6 = 1234$, $t_7 = 15$, $t_8 = 25$, $t_9 = 125$, $u_0 = 35$, $u_1 = 135$, $u_2 = 235$, $u_3 = 1235$, $u_4 = 145$, $u_5 = 245$, $u_6 = 1245$, $u_7 = 345$, $u_8 = 1345$, $u_9 = 2345$, $v_0 = 12345$
31	2^{31-26}_{III}	6 = 12, 7 = 13, 8 = 23, 9 = 123, $t_0 = 14$, $t_1 = 24$, $t_2 = 124$, $t_3 = 34$, $t_4 = 134$, $t_5 = 234$, $t_6 = 1234$, $t_7 = 15$, $t_8 = 25$, $t_9 = 125$, $u_0 = 35$, $u_1 = 135$, $u_2 = 235$, $u_3 = 1235$, $u_4 = 45$, $u_5 = 145$, $u_6 = 245$, $u_7 = 1245$, $u_8 = 345$, $u_9 = 1345$, $v_0 = 2345$, $v_1 = 12345$

Note: No main effect or two-factor interaction is clear for the designs in this table.

Table 5A.5 Sixty-Four Run 2^{k-p} Fractional Factorial Designs ($k - p = 6$, $7 \leq k \leq 17$) (k is the number of factors and F & R is the fraction and resolution)

<i>k</i>	F & R	Design Generators	Clear Effects
7	2^{7-1}_{VII}	7 = 123456	All 21 2fi's
8	2^{8-2}_V	7 = 1234, 8 = 1256	All 28 2fi's
9	2^{9-3}_{IV}	7 = 123, 8 = 1245, 9 = 1346	All 2fi's except 12, 13, 17, 23, 27, 37
10	2^{10-4}_{IV}	7 = 123, 8 = 1245, 9 = 1246, $t_0 = 1356$	All 2fi's except 12, 13, 17, 23, 27, 37, 56, 58, 59, 68, 69, 89
11	2^{11-5}_{IV}	7 = 123, 8 = 124, 9 = 1345, $t_0 = 1346$, $t_1 = 1256$	The 10 2fi's involving t_1 and the 24 2fi's between any of the factors 1, 2, 3, 4, 7, 8 and any of the factors 5, 6, 9, t_0
12	2^{12-6}_{IV}	7 = 123, 8 = 124, 9 = 1345, $t_0 = 1346$, $t_1 = 1256$, $t_2 = 23456$	The 36 2fi's between any of the factors 1, 2, 3, 4, 7, 8 and any of the factors 5, 6, 9, t_0 , t_1 , t_2
13	2^{13-7}_{IV}	7 = 123, 8 = 124, 9 = 135, $t_0 = 145$, $t_1 = 236$, $t_2 = 2456$, $t_3 = 3456$	The 20 2fi's between any of the factors 4, 5, 8, 9, t_0 and any of the factors 6, t_1 , t_2 , t_3

Table 5A.5 Sixty-Four Run 2^{k-p} Fractional Factorial Designs ($k - p = 6$, $7 \leq k \leq 17$) (k is the number of factors and F & R is the fraction and resolution)

k	F & R	Design Generators	Clear Effects
13	2^{13-7}_{IV}	$7 = 123, 8 = 124, 9 = 134,$ $t_0 = 2345, t_1 = 2346, t_2 = 156,$ $t_3 = 123456$	The 36 2fi's between any of the factors 2, 3, 4, 7, 8, 9 and any of the factors 5, 6, t_0, t_1, t_2, t_3
14	2^{14-8}_{IV}	$7 = 123, 8 = 124, 9 = 125,$ $t_0 = 2345, t_1 = 136, t_2 = 146,$ $t_3 = 156, t_4 = 3456$	$1t_0, 1t_4, 3t_0, 3t_4, 4t_0, 4t_4, 5t_0, 5t_4$
14	2^{14-8}_{IV}	$7 = 123, 8 = 124, 9 = 134,$ $t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 145, t_4 = 2356$	The 25 2fi's that involve either factor 6 or t_4 or both
15	2^{15-9}_{IV}	$7 = 123, 8 = 124, 9 = 125,$ $t_0 = 2345, t_1 = 136, t_2 = 146,$ $t_3 = 156, t_4 = 3456, t_5 = 123456$	None
15	2^{15-9}_{IV}	$7 = 123, 8 = 124, 9 = 134,$ $t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 2456$	The 27 2fi's that involve either factor 6 or t_5 or both
16	2^{16-10}_{IV}	$7 = 123, 8 = 124, 9 = 134,$ $t_0 = 125, t_1 = 135, t_2 = 126,$ $t_3 = 136, t_4 = 1456, t_5 = 2456,$ $t_6 = 3456$	None
16	2^{16-10}_{IV}	$7 = 123, 8 = 124, 9 = 134,$ $t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 245, t_6 = 3456$	The 29 2fi's that involve either factor 6 or t_6 or both
17	2^{17-11}_{IV}	$7 = 123, 8 = 124, 9 = 134,$ $t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 126, t_4 = 136, t_5 = 1456,$ $t_6 = 2456, t_7 = 3456$	None
17	2^{17-11}_{IV}	$7 = 123, 8 = 124, 9 = 134,$ $t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 245,$ $t_6 = 345, t_7 = 123456$	The 31 2fi's that involve either factor 6 or t_7 or both

Note: Because the designs in this table have at least resolution IV, all their main effects are clear and will not be repeated in the column "Clear Effects."

Table 5A.6 Sixty-Four Run 2^{k-p} Fractional Factorial Designs ($k - p = 6$, $18 \leq k \leq 32$) (k is the number of factors and F & R is the fraction and resolution)

k	F & R	Design Generators
18	2^{18-12}_{IV}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 126, t_5 = 136, t_6 = 1456, t_7 = 2456, t_8 = 3456$
19	2^{19-13}_{IV}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 126, t_5 = 136, t_6 = 236, t_7 = 1456, t_8 = 2456,$ $t_9 = 3456$

(continued)

Table 5A.6 (Continued)

k	F & R	Design Generators
20	2_{IV}^{20-14}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 126, t_5 = 136, t_6 = 236, t_7 = 1456, t_8 = 2456,$ $t_9 = 3456, u_0 = 123456$
21	2_{IV}^{21-15}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 126, t_6 = 146, t_7 = 246, t_8 = 156,$ $t_9 = 356, u_0 = 456, u_1 = 23456$
22	2_{IV}^{22-16}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 126, t_6 = 136, t_7 = 146, t_8 = 246,$ $t_9 = 156, u_0 = 356, u_1 = 456, u_2 = 23456$
23	2_{IV}^{23-17}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 245, t_6 = 126, t_7 = 136, t_8 = 146,$ $t_9 = 346, u_0 = 156, u_1 = 356, u_2 = 456, u_3 = 23456$
24	2_{IV}^{24-18}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 245, t_6 = 126, t_7 = 136, t_8 = 236,$ $t_9 = 146, u_0 = 246, u_1 = 156, u_2 = 356, u_3 = 456,$ $u_4 = 23456$
25	2_{IV}^{25-19}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 245, t_6 = 345, t_7 = 126, t_8 = 136,$ $t_9 = 236, u_0 = 146, u_1 = 246, u_2 = 156, u_3 = 356,$ $u_4 = 456, u_5 = 23456$
26	2_{IV}^{26-20}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 245, t_6 = 345, t_7 = 126, t_8 = 136,$ $t_9 = 236, u_0 = 146, u_1 = 246, u_2 = 346, u_3 = 156,$ $u_4 = 256, u_5 = 356, u_6 = 456$
27	2_{IV}^{27-21}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 245, t_6 = 345, t_7 = 12345, t_8 = 126,$ $t_9 = 136, u_0 = 236, u_1 = 146, u_2 = 246, u_3 = 346,$ $u_4 = 156, u_5 = 256, u_6 = 356, u_7 = 456$
28	2_{IV}^{28-22}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 245, t_6 = 345, t_7 = 12345, t_8 = 126,$ $t_9 = 136, u_0 = 236, u_1 = 146, u_2 = 246, u_3 = 346,$ $u_4 = 12346, u_5 = 156, u_6 = 256, u_7 = 356, u_8 = 456$
29	2_{IV}^{29-23}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 245, t_6 = 345, t_7 = 12345, t_8 = 126,$ $t_9 = 136, u_0 = 236, u_1 = 146, u_2 = 246, u_3 = 346,$ $u_4 = 12346, u_5 = 156, u_6 = 256, u_7 = 356, u_8 = 12356,$ $u_9 = 456$
30	2_{IV}^{30-24}	$7 = 123, 8 = 124, 9 = 134, t_0 = 234, t_1 = 125, t_2 = 135,$ $t_3 = 235, t_4 = 145, t_5 = 245, t_6 = 345, t_7 = 12345, t_8 = 126,$ $t_9 = 136, u_0 = 236, u_1 = 146, u_2 = 246, u_3 = 346,$ $u_4 = 12346, u_5 = 156, u_6 = 256, u_7 = 356, u_8 = 12356,$ $u_9 = 456, v_0 = 12456$

Table 5A.6 (Continued)

<i>k</i>	F & R	Design Generators
31	2_{IV}^{31-25}	7 = 123, 8 = 124, 9 = 134, $t_0 = 234$, $t_1 = 125$, $t_2 = 135$, $t_3 = 235$, $t_4 = 145$, $t_5 = 245$, $t_6 = 345$, $t_7 = 12345$, $t_8 = 126$, $t_9 = 136$, $u_0 = 236$, $u_1 = 146$, $u_2 = 246$, $u_3 = 346$, $u_4 = 12346$, $u_5 = 156$, $u_6 = 256$, $u_7 = 356$, $u_8 = 12356$, $u_9 = 456$, $v_0 = 12456$, $v_1 = 13456$
32	2_{IV}^{32-26}	7 = 123, 8 = 124, 9 = 134, $t_0 = 234$, $t_1 = 125$, $t_2 = 135$, $t_3 = 235$, $t_4 = 145$, $t_5 = 245$, $t_6 = 345$, $t_7 = 12345$, $t_8 = 126$, $t_9 = 136$, $u_0 = 236$, $u_1 = 146$, $u_2 = 246$, $u_3 = 346$, $u_4 = 12346$, $u_5 = 156$, $u_6 = 256$, $u_7 = 356$, $u_8 = 12356$, $u_9 = 456$, $v_0 = 12456$, $v_1 = 13456$, $v_2 = 23456$

Note: The designs in this table have resolution IV; all their main effects are clear but none of their two-factor interactions are clear.

Table 5A.7 128 Run 2^{k-p} Minimum Aberration Fractional Factorial Designs ($k - p = 7, 8 \leq k \leq 14$) (k is the number of factors and F & R is the fraction and resolution)

<i>k</i>	F & R	Design Generators	Clear Effects
8	2_{VIII}^{8-1}	8 = 1234567	All 8 main effects, all 28 2fi's
9	2_{VI}^{9-2}	8 = 13457, 9 = 12467	All 9 main effects, all 36 2fi's
10	2_V^{10-3}	8 = 3456, 9 = 13457, $t_0 = 12467$	All 10 main effects, all 45 2fi's
11	2_V^{11-4}	8 = 3456, 9 = 13457, $t_0 = 12467$, $t_1 = 2567$	All 11 main effects, all 55 2fi's
12	2_{IV}^{12-5}	8 = 145, 9 = 1236, $t_0 = 2467$, $t_1 = 3567$, $t_2 = 123457$	All 12 main effects, all 2fi's except 14, 15, 18, 45, 48, 58
13	2_{IV}^{13-6}	8 = 12345, 9 = 1236, $t_0 = 124567$, $t_1 = 134567$, $t_2 = 2347$, $t_3 = 567$	All 13 main effects, all 2fi's except 23, 2 t_0 , 2 t_1 , 3 t_0 , 3 t_1 , t_0t_1 , 56, 57, 5 t_3 , 67, 6 t_3 , 7 t_3
14	2_{IV}^{14-7}	8 = 123, 9 = 456, $t_0 = 1245$, $t_1 = 1346$, $t_2 = 12467$, $t_3 = 13567$, $t_4 = 23457$	All 14 main effects, all 2fi's except 12, 13, 18, 23, 28, 38, 45, 46, 49, 56, 59, 69, 7 t_2 , 7 t_3 , 7 t_4 , t_2t_3 , t_2t_4 , t_3t_4

APPENDIX 5B: TABLES OF 2^{k-p} FRACTIONAL FACTORIAL DESIGNS IN 2^q BLOCKS

Note. Two-factor interactions are abbreviated as 2fi's.

Table 5B.1 Eight-Run 2^{k-p} Fractional Factorial Designs in 2^q Blocks
 ($k - p = 3, 4 \leq k \leq 6$) (k = number of factors, 2^{k-p} = number of runs,
 2^q = number of blocks)

k	p	q	Design Generators	Block Generators	Clear Effects
4	1	1	4 = 123	$B_1 = 12$	All four main effects
4	1	2	4 = 123	$B_1 = 12, B_2 = 13$	All four main effects
5	2	1	4 = 12, 5 = 13	$B_1 = 23$	None
6	3	1	4 = 12, 5 = 13, 6 = 23	$B_1 = 123$	None

Table 5B.2 Sixteen-Run 2^{k-p} Fractional Factorial Designs in 2^q Blocks
 ($k - p = 4, 5 \leq k \leq 9$) (k = number of factors, 2^{k-p} = number of runs,
 2^q = number of blocks)

k	p	q	Design Generators	Block Generators	Clear Effects
5	1	1	5 = 1234	$B_1 = 12$	All five main effects,
					all 2fi's except 12
5	1	2	5 = 1234	$B_1 = 12,$ $B_2 = 13$	All five main effects, 14, 15, 24, 25, 34, 35, 45
5	1	3	5 = 123	$B_1 = 14,$ $B_2 = 24, B_3 = 34$	All five main effects
6	2	1	5 = 123, 6 = 124	$B_1 = 134$	All six main effects
6	2	1	5 = 12, 6 = 134	$B_1 = 13$	3, 4, 6, 23, 24, 26, 35, 45, 56
6	2	2	5 = 123, 6 = 124	$B_1 = 134,$ $B_2 = 234$	All six main effects
6	2	2	5 = 12, 6 = 134	$B_1 = 13,$ $B_2 = 14$	3, 4, 6, 23, 24, 26, 35, 45, 56
6	2	3	5 = 123, 6 = 124	$B_1 = 13,$ $B_2 = 23, B_3 = 14$	All six main effects
7	3	1	5 = 123, 6 = 124, 7 = 134	$B_1 = 234$	All seven main effects
7	3	2	5 = 123, 6 = 124, 7 = 134	$B_1 = 12,$ $B_2 = 13$	All seven main effects
7	3	3	5 = 123, 6 = 124, 7 = 134	$B_1 = 12,$ $B_2 = 13, B_3 = 14$	All seven main effects

Table 5B.2 (Continued)

k	p	q	Design Generators	Block Generators	Clear Effects
8	4	1	5 = 123, 6 = 124, 7 = 134, 8 = 234	$B_1 = 12$	All eight main effects
8	4	2	5 = 123, 6 = 124, 7 = 134, 8 = 234	$B_1 = 12,$ $B_2 = 13$	All eight main effects
8	4	3	5 = 123, 6 = 124, 7 = 134, 8 = 234	$B_1 = 12,$ $B_2 = 13, B_3 = 14$	All eight main effects
9	5	1	5 = 12, 6 = 13, 7 = 14, 8 = 234, 9 = 1234	$B_1 = 23$	None
9	5	2	5 = 12, 6 = 13, 7 = 14, 8 = 234, 9 = 1234	$B_1 = 23, B_2 = 24$	None

Table 5B.3 Thirty-Two Run 2^{k-p} Fractional Factorial Designs in 2^q blocks ($k - p = 5, 6 \leq k \leq 9$) (k = number of factors, 2^{k-p} = number of runs, 2^q = number of blocks)

k	p	q	Design Generators	Block Generators	Clear 2fi's
6	1	1	6 = 12345	$B_1 = 123$	All 15 2fi's
6	1	2	6 = 12345	$B_1 = 134,$ $B_2 = 234$	All 2fi's except 12
6	1	3	6 = 12345	$B_1 = 135,$ $B_2 = 235,$ $B_3 = 145$	All 2fi's except 12, 34, 56
6	1	4	6 = 12345	$B_1 = 12,$ $B_2 = 13,$ $B_3 = 14, B_4 = 15$	None
7	2	1	6 = 123, 7 = 1245	$B_1 = 134$	All 2fi's except 12, 13, 16, 23, 26, 36
7	2	2	6 = 123, 7 = 1245	$B_1 = 134,$ $B_2 = 234$	All 2fi's except 12, 13, 16, 23, 26, 36
7	2	3	6 = 123, 7 = 1245	$B_1 = 234,$ $B_2 = 235,$ $B_3 = 1345$	14, 15, 17, 24, 25, 27, 34, 35, 37, 46, 56, 67
7	2	4	6 = 123, 7 = 145	$B_1 = 12,$ $B_2 = 13,$ $B_3 = 14, B_4 = 15$	None
8	3	1	6 = 123, 7 = 124, 8 = 1345	$B_1 = 125$	15, 18, 25, 28, 35, 38, 45, 48, 56, 57, 58, 68, 78
8	3	2	6 = 123, 7 = 124, 8 = 1345	$B_1 = 13, B_2 = 14$	15, 18, 25, 28, 35, 38, 45, 48, 56, 57, 58, 68, 78

(continued)

Table 5B.3 (Continued)

k	p	q	Design Generators	Block Generators	Clear 2fi's
8	3	3	6 = 123, 7 = 124, 8 = 1345	$B_1 = 13, B_2 = 23,$ $B_3 = 14$	15, 18, 25, 28, 35, 38, 45, 48, 56, 57, 58, 68, 78
8	3	4	6 = 123, 7 = 124, 8 = 135	$B_1 = 12, B_2 = 13,$ $B_3 = 14,$ $B_4 = 15$	None
9	4	1	6 = 123, 7 = 124, 8 = 134, 9 = 2345	$B_1 = 12$	15, 19, 25, 29, 35, 39, 45, 49, 56, 57, 58, 59, 69, 79, 89
9	4	2	6 = 123, 7 = 124, 8 = 134, 9 = 2345	$B_1 = 12,$ $B_2 = 13$	15, 19, 25, 29, 35, 39, 45, 49, 56, 57, 58, 59, 69, 79, 89
9	4	3	6 = 123, 7 = 124, 8 = 134, 9 = 2345	$B_1 = 12,$ $B_2 = 13,$ $B_3 = 14$	15, 19, 25, 29, 35, 39, 45, 49, 56, 57, 58, 59, 69, 79, 89
9	4	4	6 = 123, 7 = 124, 8 = 135, 9 = 145	$B_1 = 12, B_2 = 13,$ $B_3 = 14,$ $B_4 = 15$	None

Note: All the main effects are clear for all of the designs in this table and will not be repeated in the column "Clear 2fi's."

Table 5B.4 Sixty-Four Run 2^{k-p} Fractional Factorial Designs in 2^q Blocks ($k - p = 6, 7 \leq k \leq 9$) (k = number of factors, 2^{k-p} = number of runs, 2^q = number of blocks)

k	p	q	Design Generators	Block Generators	Clear 2fi's
7	1	1	7 = 123456	$B_1 = 123$	All 21 2fi's
7	1	2	7 = 123456	$B_1 = 123, B_2 = 145$	All 21 2fi's
7	1	3	7 = 123456	$B_1 = 123, B_2 = 145,$ $B_3 = 246$	All 21 2fi's
7	1	4	7 = 12345	$B_1 = 12, B_2 = 34,$ $B_3 = 135, B_4 = 16$	All 2fi's except 12, 16, 26, 34, 57
7	1	5	7 = 12345	$B_1 = 12, B_2 = 13,$ $B_3 = 14, B_4 = 15,$ $B_5 = 16$	None
8	2	1	7 = 1234, 8 = 1256	$B_1 = 135$	All 28 2fi's
8	2	2	7 = 1234, 8 = 1256	$B_1 = 135, B_2 = 246$	All 28 2fi's
8	2	3	7 = 1234, 8 = 1256	$B_1 = 146, B_2 = 246,$ $B_3 = 13456$	All 2fi's except 12, 35

Table 5B.4 (Continued)

k	p	q	Design Generators	Block Generators	Clear 2fi's
8	2	4	7 = 1234, 8 = 1256	$B_1 = 13, B_2 = 14,$ $B_3 = 25, B_4 = 26$	12, 15, 16, 17, 18, 23, 24, 27, 28, 35, 36, 37, 38, 45, 46, 47, 48, 57, 58, 67, 68
8	2	5	7 = 123, 8 = 12456	$B_1 = 12, B_2 = 13,$ $B_3 = 14, B_4 = 15,$ $B_5 = 16$	None
9	3	1	7 = 123, 8 = 1245, 9 = 1346	$B_1 = 1256$	All 2fi's except 12, 13, 17, 23, 27, 37
9	3	2	7 = 123, 8 = 1245, 9 = 1346	$B_1 = 156,$ $B_2 = 123456$	All 2fi's except 12, 13, 17, 23, 27, 37
9	3	3	7 = 123, 8 = 1245, 9 = 1346	$B_1 = 156, B_2 = 256,$ $B_3 = 356$	All 2fi's except 12, 13, 17, 23, 27, 37
9	3	4	7 = 123, 8 = 1245, 9 = 1346	$B_1 = 12, B_2 = 13,$ $B_3 = 14, B_4 = 56$	15, 16, 18, 19, 25, 26, 28, 29, 35, 36, 38, 39, 45, 46, 48, 49, 57, 58, 59, 67, 68, 69, 78, 79
9	3	5	7 = 123, 8 = 124, 9 = 13456	$B_1 = 12, B_2 = 13,$ $B_3 = 14, B_4 = 15,$ $B_5 = 16$	None

Note: All the main effects are clear for all of the designs in this table and will not be repeated in the column "Clear 2fi's."

Table 5B.5 128-Run 2^{k-p} Fractional Factorial Designs in 2^q Blocks

($k - p = 7, k = 8, 9$) (k = number of factors, 2^{k-p} = number of runs, 2^q = number of blocks)

k	p	q	Design Generators	Block Generators	Clear 2fi's
8	1	1	8 = 1234567	$B_1 = 1234$	All 28 2fi's
8	1	2	8 = 1234567	$B_1 = 1234, B_2 = 1256$	All 28 2fi's
8	1	3	8 = 1234567	$B_1 = 1234, B_2 = 1256,$ $B_3 = 1357$	All 28 2fi's
8	1	4	8 = 123456	$B_1 = 123, B_2 = 145,$ $B_3 = 246, B_4 = 17$	All 2fi's except 17

(continued)

Table 5B.5 (Continued)

k	p	q	Design Generators	Block Generators	Clear 2fi's
8	1	5	8 = 123456	$B_1 = 12, B_2 = 13,$ $B_3 = 45, B_4 = 46,$ $B_5 = 147$	All 2fi's except 12, 13, 23, 45, 46, 56, 78
8	1	6	8 = 1234567	$B_1 = 12, B_2 = 13,$ $B_3 = 14, B_4 = 15,$ $B_5 = 16, B_6 = 17$	None
9	2	1	8 = 12345, 9 = 12367	$B_1 = 1246$	All 36 2fi's
9	2	2	8 = 12345, 9 = 12367	$B_1 = 1246, B_2 = 1357$	All 36 2fi's
9	2	3	8 = 12345, 9 = 12367	$B_1 = 146, B_2 = 2346,$ $B_3 = 1357$	All 36 2fi's
9	2	4	8 = 12345, 9 = 12367	$B_1 = 12, B_2 = 134,$ $B_3 = 136, B_4 = 357$	All 2fi's except 12, 46
9	2	5	8 = 12345, 9 = 12367	$B_1 = 123, B_2 = 14,$ $B_3 = 25, B_4 = 16,$ $B_5 = 27$	All 2fi's except 14, 16, 25, 27, 38, 39, 46, 57, 89
9	2	6	8 = 12345, 9 = 12367	$B_1 = 12, B_2 = 13,$ $B_3 = 14, B_4 = 15,$ $B_5 = 16, B_6 = 17$	None

Note: All the main effects are clear for all of the designs in this table and will not be repeated in the column "Clear 2fi's."

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